Intraday Patterns in Exchange Rate of Return of the Chilean Peso: New Evidence for Day-of-the-Week Effect

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* We would like to thank Conicyt for partial funding of this paper through Fondecyt Project Number
Nº 1080382. We also had the valuable research assistantship of Victor Farías.
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Abstract

We use a new statistical test based on the signal coherence function (Hinich, 2000 and Hinich and Wild, 2001) to detect subtle periodicities in the Chilean exchange rate. We resort to a unique intraday data set that allows us to capture persistent cyclical movements during the day that challenge the random walk hypothesis. We provide a micro-structural explanation for the observed behavior, and also look at the day-of-the-week-effect for the Chilean peso and find that the different days of the week have indeed different behavior patterns. This is an important result for investment allocation and risk assessment.

JEL Classification: F31, G10, G14, G15.

Key words: foreign exchange, microstructure, emerging economy, Signal coherence, periodicities
I. Introduction

The efficient market hypothesis is the central tenet of financial economics. In an efficient market, the market price is an unbiased estimate of the true asset value. Seasonal patterns have been one of the topics of most interest for researchers in the field of finance over the recent past decades. Patterns have been observed in returns, return volatility, bid-ask spreads and trading volume in a broad range of financial data. (Brooks et al., 2006 and Harris, 1986) A deviation between the true asset value and its market price should be temporary and not a systematic relationship. If we find a consistent seasonal pattern in financial data then the efficient market hypothesis is challenged.

As seasonal patterns are examples of limit cycle oscillations they are not fixed period cycles. Periodic solutions cannot be generated by linear dynamical systems. Pure periodic cycles can only be generated by nonlinear dynamical systems. (Lefschetz, 2005 and Hartman, 2002)

Harris (1986) examined both weekly and intraday patterns in stock returns. He found an average price drop on Monday mornings, while on the other weekday mornings, the price rises. However, most of the observed day-of-the-week effects occur immediately after the opening of the market. Wood et al. (1985) examined minute-by-minute returns data for a large sample of NYSE stocks. They found that significantly positive returns were earned on average during the first 30 minutes of
trading and also at market close, a result echoed by Ding and Lau (2001) using a sample of 200 stocks from the Stock Exchange of Singapore. These studies suggest that intraday patterns are quite relevant for understanding the day-of-the-week effect. Most of the recent literature has used dummy variables or simply statistical average of grouped data. Our approach however, complements the toolkit available for researchers by providing a flexible test to detect subtle periodicities that would otherwise be unobserved if we resort to traditional methods.

There are hardly any papers in the financial literature that study intraday patterns of exchange rate of returns. Most of the papers on foreign exchange markets have been devoted to intraday volatility or bid-ask spreads, rather than returns. Notable exceptions are the works of Ranaldo (2007), Brooks and Hinich (2006), Ito (1987) and Ito and Roley (1987, 1991).

In this article we contribute to the literature presenting a study of the time series for the Chilean exchange rate (Chilean Pesos/US Dollar) using a novel technique to determine if there are any seasonal patterns in the data\(^1\). The Chilean exchange market is a good representative of many other developing countries, especially the Latin American ones, in which the exchange market is dominated by two main groups of agents. First, we have the banks that operate mandated by local firms to serve the means of payments for international transactions. They operate during the

\(^1\) There are few academic papers that focus on nonlinearity of Latin American financial data. Notable exceptions are the works of Bonilla et al (2008) and Romero-Meza et al (2007)
whole time the market is open buying and selling dollars depending on the type of firm (exporter or importer). However, when a big transaction has to be made (big for the relative size of the market) banks wait for the moment of high liquidity to avoid paying big spreads. High liquidity usually concentrates in the middle of the open market time and at the end of the day. Second, we have the AFP’s (pension funds managers) that allocate some of their funds in the international markets. AFP’s are usually big players in these developing markets, so when they reallocate their international portfolio they buy or sell dollars in the local markets originating important movements in the exchange rate. To crown all this, due to the minimum relative rate of return guarantee regulation, the herding behavior of AFP’s gives rise to a disruption such that a simple portfolio reallocation ends up with an exponential effect on the exchange rate returns (Olivares, 2008). This minimum rate of return guarantee is an institutional feature of the Chilean market that compels each one of the AFP’s to mimic the benchmark in their portfolio allocation. When a negative deviation below a certain threshold occurs, the owners of an AFP’s must infuse resources of their own personal equity into the fund in order to maintain it’s return above the legal minimum established by law (Romero-Meza, 2000).

In principle, the low transaction costs and the existence of non-delivery forwards and other derivative instruments associated with exchange markets could lead us to think that, if a market is likely to be efficient, this exchange market should probably be the one. The harsh reality however, at least in the case for Chile with its market microstructure, makes us hesitate about satisfying the random walk hypothesis.
This paper also investigates an important issue in financial economics, the day-of-the-week effect, which is one of the possible explanations for seasonal pattern in financial data. In practice, it means that knowing the particular day of the week when the trading of an asset takes place is enough to infer the asset’s expected rate of return and hence its true value. Market microstructure literature can provide the economic rationale underlying these patterns.

Early research on day-of-the-week effect is found in French (1980), Gibbons and Hess (1981), and Keim and Stambaugh (1984). It is well documented that the mean return on Monday is significantly negative, while the mean return of the rest of the days is significantly positive. However, most of this literature is developed for stock markets in the US and some European exceptions as well.

Research on day-of-the-week effect for exchange rates is more limited but exists. Brooks and Hinich (2006) document the existence of significant periodic movements in several European exchange rates and Ranaldo (2007) found a time of day pattern in the exchange rate data for some developed countries. Following, Ranaldo (2007) one plausible explanation for the day-of-the-week effect in foreign exchange markets is the effect of liquidity, which refers to imbalances in the inventories of liquidity suppliers that are caused by systematic excess demand or supply at specific intraday times. We cannot observe these in our case because we do not have several markets trading in the Chilean exchange rate that may result in overlapping transaction times that have different opening and closing times that cause liquidity imbalances as in the case of Ranaldo (2007). In Chile the main reason for the day-of-the week
effect has to do with the financial microstructure and the prevalence of institutional investors in the exchange market. Our results are in some sense contrary to those of Chan et. al. (2004) who find that seasonal effects are stronger in stocks with low institutional holdings (less sophisticated individual investors) and weaker in stocks with high institutional holdings (in principle more sophisticated and better informed agents). We find that the Chilean exchange rate has strong seasonal effects and we explain those patterns mainly on account of the high institutional holdings held by the few relevant actors in the Chilean market. Again, the minimum rate of return regulation that motivates a herding behavior among AFP’s makes it unlikely that a particular AFP should depart from a subtle cyclical movement that the rest of the competitors are experiencing. Since they all mimic the benchmark portfolio holding, then they all will make an even sharper periodic movement.

II. Methodology

In this paper we resort to a novel methodology with a view to study the existence of periodicity in the Chilean intraday exchange rate series. This methodology goes far beyond the traditional approach of calculating averages or applying dummy variables. If we have a periodic signal, we could predict it far into the future since it repeats itself at every period. However, for most time series there is some variation in the waveform over time of the series even though they seem periodic. Hinich (2000) defines and studies the properties of this kind of varying periodic signal,
known as randomly modulated periodicity (RMP). He also proposes a measure called signal coherence function for the amount of random variation in each Fourier component of the signal and develops its statistical properties. This new methodology may be used for any natural or social phenomenon where periodicity is thought to be present. Further extensions of this parametric statistical model are Hinich and Wild (2001), Hinich (2003) and Hinich and Wild (2005). Hinich and Wild (2001) develop a test of the null hypothesis in which an observed time series is a realization of a strictly stationary random process. They show that this test has considerable power against processes where the mean of a time series is periodic with random variation in its periodic structure. Hinich (2003) proposes a likelihood ratio test for RMP when the amplitudes and phases are known. Hinich and Wild (2005) propose a method for detecting an RMP, the amplitudes and phases of which are an unknown random process plus a stationary noise process.

This paper is the first application of this methodology to a financial time series in emerging capital markets. Furthermore, we complement the periodicity determination with the study of intraday patterns for five different groups. We divide the whole sample and compose each of the five series by grouping the rates of return for the same day of the week. Our approach to this topic allows us to obtain a conditional expectation of the rate of return for each day of the week that only depends on the prices observed within the group. By focusing on the marginal probability of a given day we only incur in a small efficiency cost that should not be
relevant given the large size of the dataset and the gain is the isolation of each of the five groups and the study of their behavior.

Among the different approaches to detect seasonal patterns, it is well worth stressing the differences between those techniques that search for deterministic patterns from those that search for stochastic patterns. Among the former group are the dummy variables and the grouping by hours. The Randomly Modulated Periodicity technique which affords the clear advantage of allowing a flexible specification of the exchange rate return process belongs to the latter group. The RPM technique is what we have used in this paper.

II.1A Randomly Modulated Periodicity

The parametric statistical model called randomly modulated periodicity introduced by Hinich (2000) allows detecting subtle periodicities in financial time series. This approach can be applied to any fairly large, evenly spaced sample of time series data that is thought to contain periodicities. This model is based on a Fourier analysis. The basic idea is that any periodic function of period T can be written as a sum of weighted sine and cosine functions, the frequencies of which are integer multiples of the fundamental frequency 1/T. These frequencies are called Fourier frequencies. The weights, known as amplitudes, are fixed constants for a deterministic periodic function. The sum is called a Fourier transform of the periodic function. But a perfectly periodic function is an idealization of a real periodic process.
Thus the amplitudes of the Fourier transform of a real periodic process are a constant plus a zero mean random time series that may or may not be stationary. The random time variations makes the amplitudes “wobble” over time causing the signal to have period-to-period random variation.

Hinich (2000) introduces a measure of the wobble in the Fourier amplitudes as a function of frequency. This new form of spectrum is called a signal coherence spectrum and is very different from the ordinary power spectrum. For one thing, it is a normalized statistic that is independent of the height of the power spectrum at each frequency.

A discrete-time random process \( \{ x( t_n ) \}, n = 0, 1, 2, \ldots \) is an RMP with period \( T = N \delta \), \( \delta \) is the sampling interval and \( K \) harmonic frequencies if it is of the form

\[
x(t_n) = a_0 + \frac{2}{N} \sum_{k=1}^{K} (a_{ik} + u_{ik}(t_n)) \cos(2\pi f_k t_n) + \frac{2}{N} \sum_{k=1}^{K} (a_{2k} + u_{2k}(t_n)) \sin(2\pi f_k t_n)
\]

(1)

where \( f_k = k/T \) is the \( k \)-th Fourier frequency and \( u_{ik} \) \((i=1,2)\) are jointly dependent zero mean random processes that are periodic block stationary and satisfy finite dependence. These time series are called modulations in signal processing literature. The coefficient \( a_0 \) is a constant because the fundamental and harmonic frequency components determine the modulations. Furthermore, the variability is in the modulations rather than in the additive noise, and consequently this specification would be termed a random-effects model. The data-generating mechanism produces modulations which would be deterministic, though the researchers as rule are not
aware of this on an a priori basis and consequently treat them as a random process.

Given the conditions for RMP, the modulations are approximately stationary within each period. The process \( x(t_n) \) can be written as

\[
x(t_n) = a(t_n) + u(t_n)
\]

where

\[
a(t_n) = E[x(t_n)] = a_0 + \frac{2}{N} \sum_{k=1}^{\infty} [a_{1k} \cos(2\pi f_k t_n) + a_{2k} \sin(2\pi f_k t_n)]
\]

and

\[
u(t_n) = \frac{2}{N} \sum_{k=1}^{\infty} [u_{1k} \cos(2\pi f_k t_n) + u_{2k} \sin(2\pi f_k t_n)]
\]

Thus, the expected value of the series, \( a(t_n) \), is a periodic function. The fixed coefficients \( a_{1k} \) and \( a_{2k} \) determine the shape of \( a(t_n) \). If \( a_{11} \neq 0 \) or \( a_{21} \neq 0 \) then \( a(t_n) \) is periodic with period \( T=N\delta \). If \( a_{11}=0 \) and \( a_{21} = 0 \), but \( a_{12} \neq 0 \) or \( a_{22} \neq 0 \), then \( a(t_n) \) is periodic with period \( T/2 \). If the first \( k_0-1 \) \( a_{1k} \) and \( a_{2k} \) are zero, but not the next, then \( a(t_n) \) is periodic with period \( T/ k_0 \).

**II.2 Signal Coherence Function**

Hinich (2000) introduces a concept called signal coherence function to measure the modulation relative to the underlying periodicity (SIGCOH). For each Fourier frequency \( f_k = k/T \) the value of SIGCOH is
\[ \gamma_s(k) = \sqrt{\frac{|a_k|^2}{|a_k|^2 + \sigma^2_u(k)}} \]  

(5)

where \( a_k = a_{1k} + i a_{2k} \) is the amplitude of the \( k \)th sinusoid in complex variable form, \( i = \sqrt{-1} \), \( \sigma^2_u(k) = E|U(k)|^2 \) and \( U(k) = \sum_{n=0}^{N-1} u_k(t_n) \exp(-i2\pi k t_n) \) is the discrete Fourier transform (DFT) of the modulation process \( u_k(t_n) = u_{1k}(t_n) + i u_{2k}(t_n) \) written in complex variable form. The SIGCOH measures the amount of “wobble” in each frequency component of the signal \( x(t_n) \), by assigning a coefficient in the [0,1] interval, where the extremes represents the polar cases of a pure signal \( (\gamma_s(k)=1, \ a(k) \neq 0 \) and \( \sigma^2_u(k)=0) \), and pure noise \( (\gamma_s(k)=0, \ a(k) = 0 \) and \( \sigma^2_u(k) \neq 0) \). Thus, SIGCOH is loosely analogous to the standard R\(^2\)-measure used in regression analysis. Here it quantifies the degree of association between two components for each given frequency. The amplitude-to-modulation standard deviation (AMS) is

\[ \rho_s(k) = \frac{|a_k|}{\sigma_u(k)} \]  

for frequency \( f_k \). Thus, \( \gamma_s^2(k) = \frac{\rho_s^2(k)}{\rho_s^2(k) + 1} \) is a monotonically increasing function of this signal-to-noise ratio. An AMS of 1.0 equals a signal coherence of 0.71 and an AMS of 0.5 equals a signal coherence of 0.45.

To estimate the SIGCOH, \( \gamma_s(k) \), suppose that we know the fundamental period and we observe the signal over \( M \) such periods. The \( m \)th period is \( \{x((m-1)T + t_n), n = 0,\ldots,N-1\} \). The estimator of \( \gamma_s(k) \) is

\[ \hat{\gamma}(k) = \sqrt{\frac{|\bar{X}(k)|^2}{|\bar{X}(k)|^2 + \hat{\sigma}^2_u(k)}} \]  

(6)
where \( X(k) = \frac{1}{M} \sum_{m=1}^{M} X_m(k) \) is the sample mean of the DFT and

\[
X_m(m) = \sum_{n=0}^{N-1} x((m-1)T + t_n) \exp(-i2\pi m t_n) \quad \text{and} \quad \hat{\sigma}_u^2 = \frac{1}{M} \sum_{m=1}^{M} \left| X_m(k) - \bar{X}(k) \right|^2
\]

is the sample variance of the residual discrete Fourier transform (DFT), \( X_m(k) - \bar{X}(k) \). This estimator is consistent as \( M \to \infty \) and if the modulations have a finite dependence then the distribution of \( \frac{M \left| \bar{X}(k) \right|^2}{\sigma_u^2(k)} \) is asymptotically chi-squared with two degrees of freedom and a non-centrality parameter \( \lambda_k = (M/N) \rho^2(k) \) as \( M \to \infty \) (see Hinich (2001)). These \( \chi^2_2(\lambda_k) \) variates are approximately independently distributed over the frequency band.

We also resort to a joint test of the null hypothesis that there is zero coherence across the \( M \) frames for all \( K \) frequencies examined (Hinich and Wild, 2001). This statistic test will asymptotically follow a non-central Chi-squared distribution with \( 2K \) degrees of freedom.

Based on the frequencies with coherences higher than a given threshold, it is possible to construct the Coherent Part of the Mean Frame, which is the inverse Fourier transform of the Mean Frame for those frequencies, leaving the rest zeroed. The intuition of this is to have a measure in the time domain of the part of the time series which is periodic.
III Data

The data used corresponds to actual intraday exchange rate transactions of the Chilean banks’ trading system (DATATEC). We have the volume traded and the corresponding prices for every transaction. The intraday data spans from November of 2003 to December of 2006. Given that some periods during the trading day are more liquid than others; our dataset is not evenly spaced. Therefore, we applied a filter program develop by Hinich and available from the authors upon request. This program interpolates data point during the day in order to have observations every five minutes. This methodology does not introduce any correlation to the data. We have 85 observations per day, which is equivalent to approximately 7 hours of daily trading.

IV. Empirical Results

Table 1 presents descriptive statistics for the whole sample and for each of the five groups. We observe that the mean is different for each day, with a negative value for Monday and a positive and large value for Friday. This is consistent with most day-of-the-week effect literature.
Table 1. Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Whole</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.86×10⁻³</td>
<td>-6.68×10⁻⁵</td>
<td>-8.76×10⁻⁵</td>
<td>5.08×10⁻⁵</td>
<td>1.56×10⁻⁴</td>
<td>6.47×10⁻⁵</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.16×10⁻³</td>
<td>3.92×10⁻³</td>
<td>4.22×10⁻³</td>
<td>4.18×10⁻³</td>
<td>4.22×10⁻³</td>
<td>4.27×10⁻³</td>
</tr>
<tr>
<td>Skewness</td>
<td>7.55×10⁻¹</td>
<td>3.52×10⁻¹</td>
<td>4.02×10⁻¹</td>
<td>7.15×10⁻¹</td>
<td>3.88×10⁻¹</td>
<td>1.87</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>37.3</td>
<td>37.3</td>
<td>34.6</td>
<td>34.8</td>
<td>38.6</td>
<td>40.7</td>
</tr>
<tr>
<td>Minimum</td>
<td>-7.16×10⁻²</td>
<td>-3.64×10⁻²</td>
<td>-4.52×10⁻²</td>
<td>-5.08×10⁻²</td>
<td>-7.16×10⁻²</td>
<td>-3.73×10⁻²</td>
</tr>
<tr>
<td>Maximum</td>
<td>7.24×10⁻²</td>
<td>7.16×10⁻²</td>
<td>6.84×10⁻²</td>
<td>5.32×10⁻²</td>
<td>5×10⁻²</td>
<td>7.24×10⁻²</td>
</tr>
</tbody>
</table>

After trying different frame length we select 170 observations that correspond to two trading days. We apply the Hinich and Wild (2001) test of the joint hypothesis that there is not any significant coherence across the data. The two-day frame gives a p-value that is consistent with rejecting the null hypothesis.

Table 2 presents p-values for the whole sample for each one of the days. The results show that we reject the null hypothesis for both the whole sample as well as
Tuesday at 1% significance level, and for Friday at 5% significance level. We cannot reject the null hypothesis for the rest of the days. This is evidence that the behavior of the returns of each day is different, partially supporting the day-of-the-week effect for this dataset. However, from our results we know that the information for Mondays does not help in predicting what will happen on the following Monday, which makes them different from the traditional day-of-the-week effect literature.

These results are important because they confirm that different days of the week have indeed different behaviors, but those behaviors are systematic only for some days, Tuesday and Friday in our case, implying that that we have a special day-of-the-week effect for our Chilean data.

<table>
<thead>
<tr>
<th></th>
<th>Whole Sample</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td>0.0025</td>
<td>0.44</td>
<td>0.0091</td>
<td>0.41</td>
<td>0.20</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Table 2. Joint p-value test.

We can gain more insight by looking at the components of the time series whose frequencies are significant. The process of Fourier transforming data can be
understood as decomposing it into a sum of more simple components, specifically, a weighted sum of sines and cosines. There are two weights for the sine-cosine pair at each period (or frequency, with is the inverse of the period, having a reciprocal relation). These periods are $\left\{ \frac{T}{k} \right\}_{k=1}^{K}$. The collection of significant periods captures the stable part of the time series. The magnitude of the coherence reflects the relation between the stable information that can be extracted from the data and the noise concurrent to it. Therefore, the more periodic the time series, the larger are the number of significant periods and the bigger their respective coherence coefficients.

Table 3 shows the periods where Fourier coefficients (the weights) and the modulations at that period imply a significant coherence at 5%. These results are consistent with Table 2. We observe a tendency for a larger number of significant periods, the smaller the p-value is for the joint test. The size of the coherence coefficients for individual day databases is nearly double those of the whole series.
## Table 3. Significant periodicities at 5%

<table>
<thead>
<tr>
<th>Period</th>
<th>Period in h,m</th>
<th>Coherence</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Sample</td>
<td>13.0769</td>
<td>1h 5.38m</td>
<td>0.164</td>
</tr>
<tr>
<td></td>
<td>6.8</td>
<td>34</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>6.2963</td>
<td>31.48</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>5.8621</td>
<td>29.31</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>5.4839</td>
<td>27.42</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>5.3125</td>
<td>26.56</td>
<td>0.161</td>
</tr>
<tr>
<td></td>
<td>3.8636</td>
<td>19.32</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>3.0909</td>
<td>15.45</td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td>2.9825</td>
<td>14.91</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>2.4638</td>
<td>12.32</td>
<td>0.151</td>
</tr>
<tr>
<td></td>
<td>2.3944</td>
<td>11.97</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td>2.2368</td>
<td>11.18</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>2.1795</td>
<td>10.9</td>
<td>0.154</td>
</tr>
<tr>
<td>Monday</td>
<td>3.6957</td>
<td>18.48</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>2.931</td>
<td>14.66</td>
<td>0.344</td>
</tr>
<tr>
<td>Tuesday</td>
<td>85</td>
<td>7h 5m</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td>9.4444</td>
<td>47.22</td>
<td>0.322</td>
</tr>
<tr>
<td></td>
<td>7.0833</td>
<td>35.42</td>
<td>0.355</td>
</tr>
<tr>
<td></td>
<td>6.0714</td>
<td>30.36</td>
<td>0.323</td>
</tr>
<tr>
<td></td>
<td>5.3125</td>
<td>26.56</td>
<td>0.392</td>
</tr>
<tr>
<td></td>
<td>2.7419</td>
<td>13.71</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td>2.6984</td>
<td>13.49</td>
<td>0.379</td>
</tr>
<tr>
<td></td>
<td>2.6154</td>
<td>13.08</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>2.5758</td>
<td>12.88</td>
<td>0.342</td>
</tr>
</tbody>
</table>
Based on these results, we infer that there is a richer relation between returns that belong to the same day of the week within the group than the whole sample. This is particularly true for Tuesday and Friday.

It is quite interesting that the significant periods are generally smaller than 30 minutes; there are, however, important exceptions: the 7-hour and 5-minute period of Tuesday and Thursday. This result is important and it is due to the advantage of handling the problem in the frequency domain instead of the time domain, as done

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traditionally in the financial and economics literature. The RPM allows detecting subtle periodicities that would otherwise remain undisclosed in a time domain analysis. In our case, these subtle periodicities explain the high number of small significant periods. In the appendix we present the coherence of all periods through a graphical representation.

Figure 1 presents the coherent part of the mean frame (COPAM) for the whole sample, and for Tuesday and Friday, which are the series evidencing periodicity according to Table 1. The COPAM is calculated with a coherence threshold of 0.1. This representation affords an idea as to the periodicity of exchange rate returns in the time domain for the selected period, in our case two trading days. Given that the large number of frequencies that carry information are high frequency we observe high “wobbling” in these figures. We also observe two sub-periods of high volatility for each trading day (85 observations). One of them is in the middle of the trading day and the second at the end of the day. Though these patterns are common for the three series, it does not mean that they are identical.
These results confirm that intraday patterns are present in the Chilean exchange rate series and that, therefore, there is a potential to study the patterns and generate trading rules that outperform markets returns, rejecting the random walk hypothesis.

This finding makes it even more likely to find either a profitable strategy or a trading rule if we can understand the patterns embedded in the series and then exploit it using the day of the week effect information captured by the method.

Figure 2 shows the 5-minute cumulated expected returns for two trading days, calculated from the accumulation of the Coherent Part of the Mean Frame returns.
Different expected returns are found for each group of days in Chilean foreign exchange currency. Mondays and Tuesdays expected returns follow on average a negative trend while Thursdays and Fridays expected returns follow on average a positive trend, which confirm the different daily behavior found in the literature.

This evidence of day-of-the-week effect is mainly based on subtle intraday periodicities for the Chilean exchange rate market captured by the random modulated periodicity model. Our results contribute to the financial literature with
new evidence about both the day-of-the-week effect as well as the different patterns observed for each day of the week.

Further research should be conducted and some questions emerge. Can these patterns be economically exploited? If the answer is affirmative, our results would be consistent with rejecting the efficient market hypothesis.

V. Conclusions

In this article we have employed a new statistical test based on the signal coherence function to detect subtle periodicities in Chilean exchange rate. We have also analyzed the day-of-the-week effect for Chilean data and we have found interesting results.

First, significant coherence exists mainly for small frequencies in the whole sample. This confirms the existence of periodic behavior in the exchange rate data. Overall, we find the signal coherence to be maximal at the 19.32 minutes period. This opens the possibility of generating a trading rule to capture abnormal returns from the series. However, the maximal significant coherence would not be high enough yet to induce investors to adhere to such strategy. We probably need a longer sample period to better capture the periodic movements in the data and in this way be able to compare different trading rules, which is something that has to be done in future research.
We also contribute to the day-of-week literature. Instead of calculating averages or using dummy variables, we group the data by each day of the week and run a joint test to evaluate the coherence in the exchange series. Based on our results, we find a richer relation between returns belonging to the same day of the week within the group than for the whole sample. This is particularly true for Tuesday and Friday.

We found that the significant periods are generally smaller than 30 minutes, and that the most important exceptions to this were the 7-hour and 5-minute period of Tuesday and Thursday (Table 3). This result shows the advantage of handling the problem in the frequency domain instead of the time domain, as it is done traditionally in financial and economics literature. The RPM allows detecting subtle periodicities that remain undetected in a time domain analysis. In our case, these subtle periodicities are the explanation of the high number of small significant periods.

The results show significant coherence for Tuesday and Friday with coherence near to 0.4 in each case. In particular, we observe that there is a negative cumulative return on Tuesday and a positive cumulative return on Friday. We cannot say which are the exact reasons for this behavior; it is only a matter of conjecture. However, we believe that given the coherence found in the grouping exercise, this can actually be of more interest for sophisticated traders since the smaller modulation increases the probability of taking a better advantage from the periodic time series.
We have contributed to the literature by applying a new statistical technique to an emerging market with a unique data set. We observe periodic movements that contradict the random walk hypothesis for this emerging economy.
VI. References


