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SOCIAL CHOICE AND TIME CONSISTENCY WITH LOW-PROBABILITY EVENTS

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A key result in macroeconomics is the "time inconsistency of short-run optimal plans." It is argued that inflationary bias results if central bankers do not precommit to a monetary policy rule. The macro literature, however, does not address the way in which board members arrive at the "optimal choice of inflation rate." That is a matter of a micro subfield called social choice. If we consider that on any board, members have different priors about the states of nature for the economy, but they all receive the same signal before deciding, then they will have different posterior probabilities for the states, even if they have many data, if one state has a *low probability* of occurring, such as an unlikely catastrophic-risk event. Thus, it is not clear what the optimal plan is. Therefore, discretion rather than rules may be the optimal plan in social choice settings.

Keywords: Social Choice, Time Consistency, Low-Probability Event

1. INTRODUCTION

Assessing risk is one of the most difficult tasks in life. In this paper, I tackle the problem of collective determination of a policy risk by a committee whose institutional responsibility is to make such a decision. I use a story about a central bank policy committee that has to decide on the level of reserves for its member banks to present a simplified problem where the committee members face a collective choice problem for the group decision on controlling bank reserve risk. Then I discuss the implications of different rules for making a group decision when a social choice cycle exists.

This approach has not being taken into account in the macro literature. Modern macro models are usually built using a representative agent or an overlapping-generations setup, which is useful in addressing economic growth or monetary issues. In both cases, however, the central bank (or the government in the fiscal policy case) makes its decision, then the private sector optimizes its behavior, and equilibrium is reached (if it happens). How does the central bank operate in

This paper originated from conversations between the author and Mel Hinich, and it can be considered a continuation of Melvin Hinich (2003), Risk when some states are low probability events. *Macroeconomic Dynamics* 7, 636–643. I am heavily in debt to Mel for his friendship and guidance. Mel rests in peace. Address correspondence to: Claudio A. Bonilla, School of Economics and Business, University of Chile, Diagonal Paraguay 257, Suite 1905, Santiago de Chile, Chile; e-mail: cbonilla@fen.uchile.cl.

practice? How does the board arrive at a decision? The answers to those questions are not in general of interest from a macro perspective. My point, however, is that if we consider that level of detail, then the traditional macro models and their policy implications may be challenged, as I will do in this paper.

My main point is that in a setting in which members of a committee have probabilistic beliefs about the future states of the world and update their beliefs in a Bayesian manner, if committee members start from different priors, even if they all receive the same signal (new information), in no way will they end up in agreement about the updated (posterior) distribution of beliefs. Thus, in many cases, it will be difficult to agree on a specific policy rule, and therefore, discretion rather than rules may be the only way out in some probabilistic social choice problems.

2. A BANK RESERVE POLICY SCENARIO

Imagine a policy committee of a central bank that has to decide to continue or change the existing level of reserves for its member banks in a fractional reserve banking system. Each committee member wishes to set a policy that provides as much borrowing capacity as possible for the member banks provided that the risk level of the system is kept under control to ensure the stability of the system.

Let L be the stochastic loss ratio relative to a bank's capitalization. If the member bank control policies are sufficiently homogeneous and stationary over time, then the banks can be considered a single population (similar to the representative agent), where L has a cumulative distribution function F_L that is independent of time.

I formalize the decision problem as follows. The committee selects a loss probability p and then selects a reserve threshold T_p relative to the bank's capitalization such that the probability that L is larger than T_p is p ; that is, $1 - F_L(T_p) = p$. No decision is needed if the committee does not change the probability p.

Suppose now that the committee decides to reduce the loss probability to a new level that we continue to denote as p for simplicity. Assume that the committee chair is empowered to select the method for determining $F_L(T_p)$ from data on relative loss ratios from member banks over time. Any method that is used is a problem in statistical estimation and inference, and that statistical problem is a central part of actuarial science, a branch of mathematical statistics.

In this story, the committee chair decides to employ a well-established firm of licensed actuaries rather than the central bank's econometric staff. The actuaries make several core assumptions based on an analysis of the database. Assumptions such as the data on loss ratios in the member bank population are assumed to reflect a random sample of the whole population. If the data represent the whole population, then the statistical trick is to assume that the actual population is a sample of an infinite population of member banks over a long time, assuming distributional stationarity to justify averaging. Recall that the statistical problem is to solve the equation $F_L(T_p) = 1 - p$ using assumptions on the F_L and the data sample.

Actuaries propose the quantile approach for estimating T_p from the sample [David and Nagaraja (2004)]. The quantile estimate $\hat{T}_p$ is the $(1 - p)$th sample quantile computed from the order statistics of the data. If (1) there are enough large values of L in the sample so that the $(1 - p)$th sample quantile is defined and (2) the sample size N is large enough to provide some confidence in the use of the large sample statistical properties of the sample quantiles, then the distribution of $\hat{T}_p$ is approximately normally distributed with mean T_p and variance $p(1-p)/N[f(T_p)]^2$. Experienced data-oriented statisticians understand that the term "large enough" for sample sizes is a judgment call and cannot be made precise without assuming a special form of the density function and precise values of the parameters of the distribution in question.

Because each member faces a choice of an outcome from the stochastic estimate based on the data, members have to choose thresholds based on their utility functions for the range of most likely thresholds given the estimation method. If each behaves as conventional statisticians tend to do, then they will all choose the modal value $\hat{T}_p$ as their most preferred threshold. However, if they differ in their risk acceptance, then their preference orderings will differ.

For the sake of argument, suppose that all the members accept the estimate $\hat{f}(T_p)$ developed by the actuaries as the best guess for $f(T_p)$, as well as the large sample approximation for the distribution of the estimate $\hat{T}_p$. Then they all share the same uncertainty about the true value of the threshold.

Before going into the case of the distribution of beliefs with the Bayesian update, I present a potential group decision problem of the committee policy choice for the threshold when the members have different risk preferences for the potential true value of the threshold.

3. GROUP CHOICE OF A THRESHOLD UNDER DIFFERENCES IN RISK ACCEPTANCE

To simplify the exposition, suppose that the committee consists of only three members, called imaginatively ONE, TWO, and THREE. Suppose that ONE prefers $A = \hat{T}_p$. Suppose that THREE prefers a more stringent threshold, $B = \hat{T}_p + \sqrt{p(1-p)/N}\hat{f}(T_p)$, one standard deviation about the mode. In contrast, TWO prefers the more relaxed threshold $C = \hat{T}_p - \sqrt{p(1-p)/N}\hat{f}(T_p)$. Assume that the committee agrees to choose among these three most preferred thresholds and that the preference profile of the committee is the classical Condorcet example in which individual preferences are transitive (rational) as shown in Table 1. Then there is a majority rule cycle for this profile, so no agreement can be achieved.

Not all three-member preferences profiles are cyclical. If THREE's transitive ordering is B > A > C, then the majority rule preference A > B > C is transitive and A is the Condorcet winner. The critical point I wish to make is that preference differences among members open up the possibility for group intransitivities.

TABLE 1. Condorcet cycle

	ONE	TWO	THREE
First choice	A	C	B
Second choice	B	A	C
Third choice	C	B	A

If we evoke the assumptions made in the Arrow theorem [Arrow (1963)], then there is no general transitive aggregation solution to the group decision problem when risk preferences differ.

Thus, even if all committee members have the same prior and they receive the same signal, if group member have different risk acceptances, they can all disagree, providing space for discretion rather than rules once again, depending on the social choice mechanism defined to select the policy (in our case, the policy of the reserve ratio for the banks).

4. GROUP CHOICE WITH DIFFERENT PRIORS AND TIME CONSISTENCY

Rules rather than policy discretion are what central bankers should follow to avoid the inconsistency of optimal short-run temptation. At least, that is what the Nobel laureates Kydland and Prescott (1977) argue in their famous and influential paper. However, to what extent can such a theoretical idea really be put in practice under social choice? The rules rather than discretion result is logically perfect in the representative agent setup with a short-run game embedded in it. However, does the representative agent correctly represent a central bank committee or board?

The representative agent has been extensively used in the macroeconomic literature since the influential work of Lucas (1976). These models assume that a single individual represents all the consumers in the economy and, at the same time, owns the representative firm. In spite of the well-known preference aggregation problems [Kirman (1992)], the representative agent model has been useful to study long-run public policy and economic growth. However, I ask again, can the dynamic decisions of a group be well represented by an agent? I think that, in group decision problems, they cannot.

A group decision is a collective choice. Therefore, social choice theory should be involved in the discretion-rule policy dilemma, but that is not what we see in traditional macro models. In consequence, we think that there is some space for improvement, to reduce the gap between classical central banks economic theory and real group decision making under probabilistic beliefs.

Let us start by taking an example found in Hinich (2003). Suppose that a Bayesian central bank committee has to decide on the level of bank reserves [this story also fits perfectly the examples of deciding on an inflation rate or choosing an investment-tax-credit, found in Kydland and Prescott (1977), or even the more recent macro literature that deals with policy games, such as Albonico and Rossi (2015)]. This decision depends on the committee members´ personal assessment of the future. The committee receives new information (the exact same information for everyone) about the economic conditions. This new information is processed by committee members, generating an updated belief about the probability distribution of the future state of the world. If this procedure repeats many times, the posterior distribution of beliefs converge to the same unique updated distribution, which is the idea behind the representative agent model and also the basis of an important branch of the literature on game theory based on the idea of common knowledge [Aumann (1976)].

Now, let us suppose that the arrival of new data happens each hour (even though we know that most of the economic information required by a central bank committee probably comes monthly or quarterly); this means that we update our posterior distribution of beliefs 8,760 times a year. If there exists a low-probability event k_0, let us say with probability of occurrence $\pi(k_0) = 10^{-5}$, then this event will occur, on the average, every 11.42 years, based on the hourly influx of new information. Therefore, nine occurrences of state k_0 occur in a 100-year period, and the sample proportion estimate of $\pi(k_0)$ is $\hat{\pi}(k_0) = 9/876,000 = 1.03 \times 10^{-5}$. The 95% approximate confidence interval is $(0.36 \times 10^{-5}, 1.7 \times 10^{-5})$. This interval contains the true value of $\pi(k_0)$ but with a 67% plus or minus spread. It will take approximately 100 years for committee members to observe sufficient outcomes of this low-probability state to estimate the probability with accuracy of an order of magnitude. The problem with this time is that nobody stays on a central bank board for 100 years. Therefore, in practice, committee members cannot converge to the same belief about the assessments of future states of the world, and in consequence, it is highly likely that they will not always agree, which means that discretion rather than rules may again be the only way to make decisions in this social choice setting, and therefore, changes in policy rules should be expected [Baxa et al. (2014)].

5. CONCLUSIONS

I have studied probabilistic decision making by groups, using an example of a central bank committee that updates its beliefs using a Bayesian approach. I have shown that when we consider the mechanisms through which the committee makes decisions, under certain circumstances such as differences in the risk acceptance of committee members or the existence of low-probability events under different priors about the probability distribution of future states of the world, important macroeconomics ideas can be challenged. In particular, I have challenged the concept of "rules rather than discretion" policy recommendations, an idea that is generally accepted by central banks around the world, and, as we have just seen, that may not be the optimal plan when social choice group decision making is in place.

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