Revisiting the Competitive Firm under Price Uncertainty

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Abstract

This paper provides a new look into the theory of the competitive firm under price uncertainty. We revisit the conditions under which a mean-preserving increase in risk translates into changes in the level of the optimal output and connect those results with recent developments in the mean-variance approach like the elasticity of risk aversion of Battermann et al. (2002). We also relate our analysis with the utility premium of Friedman and Savage (1948) and provide a fresh economic intuition for that classic result.

Keywords: Firms, Price uncertainty, (μ, σ) preferences, Utility premium, Partial prudence.

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1 Introduction

The study of the theory of the firm under uncertainty has a long history. This theory was first developed in the classic works of Baron (1970, 1971), Leland (1972) and Horowitz (1970) among others. A key idea of this theory is that under price uncertainty, an entrepreneur's output decisions and his responses to changes in uncertainty parameters are significantly influenced by his attitudes toward risk.

Sandmo (1971) studied a firm's behavior under price uncertainty and showed that if preferences exhibit decreasing absolute risk aversion (DARA), when there is price uncertainty, the firm's optimal output in a competitive market is less than the optimal output in the certainty case with the same expected price. This result is also found in Hawawini (1978), who represents firm's behavior under uncertainty in a mean-standard deviation framework and provides a clear graphical intuition that connects results from the expected utility model with the mean-standard deviation framework. Lippman and McCall (1981) also analyzed the effect of a riskier price on the choice of output of a competitive firm, allowing the price to undergo a mean-preserving increase in risk. They conclude that the output decreases or increases depending on the concavity or convexity of the function $g(t) = tu'(t)$, together with some other conditions about the marginal and average cost of a firm's output.

This classic line of work is based on very simple risk structures but provides a clear economic intuition of different comparative static exercises in risky environments. Recent developments in the literature of risk and uncertainty are focused on more complex risk structures and their axiomatic characterization. Among this new wave of models we find the works on ambiguity (Klibanoff et al., 2005, Machina, 2014) and the works on higher order risk structures (Deck and Schlesinger, 2014, Eeckhoudt et al., 2009) among others. These new lines of work are interesting because they provide the general conditions of equilibrium allocations in more complex risky settings. However, the usual trade-off in economics tell us that sometimes, the complexity of these new models makes it more difficult to infer the basic economic intuition and the comparative static results of these problems.

More recent literature also studies the firm in a context of uncertainty. Simpson and Sproule (2000) study the production responses of a competitive firm to first, second and third-order stochastic dominant shifts. Paulsson and Sproule (2002) apply stochastic dominant shifts to the case of logarithmic and positive-power utility functions to study the effects on output. Lately, regret-aversion utility functions have been incorporated into the analysis of the firm under price uncertainty with interesting results that challenge the usual knowledge about optimal production under uncertainty. (see, for example, Niu et al., 2014 or Wong, 2014).

Our paper has three goals. First, we take a new look at the theory of the firm under uncertainty to study how a mean-preserving increase in risk induces changes in the optimal output and show that the theoretical foundation of this result can be derived by comparing the solutions in the case of certainty to the case of uncertainty. Second, we characterize our results in terms of the mean-standard deviation approach, and to meet these goals, we develop a proposition in the (μ, σ) space that is connected with the results of the previous section. Also, in this section we revisit the concept of the elasticity of risk aversion developed by Batterman et al. (2002) and relate it with our results. Third, we connect these results with the utility premium of Friedman and Savage (1948) and show that output changes depend on the utility premium being decreasing or increasing in output, thereby providing a fresh look at this classic result.

2 The Model

Consider a risk-averse entrepreneur who owns a competitive firm. Let $F(p, r)$ denote the cumulative distribution function of the random output-price defined over a support in the closed interval $[a, b]$. The expectation of p is defined as $E(p) = \int_a^b pf(p, r)dp \equiv \mu_p$, where $f(p, r)$ denotes the probability density function of p, and r is a shift parameter (as in Diamond and Stiglitz, 1974) that will allow us to represent a mean-preserving increase in risk. The standard deviation of p is denoted by σ_p. The firm's profits are given by:

$$\pi(p, q) = pq - C(q) \tag{1}$$

where $C(q)$ is the increasing and convex total cost function of q units of the product or output and $\pi(p, q)$ is a concave function of q. Also, the expected profits are a function of the expected price (μ_p), that is, $E(\pi(p, q)) = \mu_p q - C(q) \equiv \mu_\pi$.

The risk-averse entrepreneur has a strictly concave utility function u that is differentiable at least 3 times. The entrepreneur seeks the optimal output q^* that maximizes the expected utility U derived from profits. Thus, the output decision for a risk-averse entrepreneur is as follows:

$$U(q^*(r)) = \max_{q>0} U(q) \tag{2}$$

where

$$U(q) = Eu(\pi(p, q)) = \int_a^b u(\pi(p, q))dF(p, r) = Eu(pq - C(q)) \tag{3}$$

The first and second-order conditions of the entrepreneur are:

$$U'(q^*(r)) = E(p - C'(q^*))u'(\pi(p, q^*)) = 0 \tag{4}$$

$$U''(q^*(r)) = E(p - C'(q^*))^2 u''(\pi(p, q^*)) - Eu'(\pi(p, q^*))C''(q^*) < 0 \tag{5}$$

Now, we make use of the following theorem.

Theorem 1 (Diamond and Stiglitz, 1974)**.** *Let $\alpha^*(r)$ be the level of the control variable which maximizes $\int_a^b u(\theta, \alpha)dF(\theta, r)$. If increases in r represent mean preserving increases in risk, then α^* increases (decreases) with r if u_α is strictly convex (concave) function of θ, i.e., if $u_{\alpha\theta\theta} > (<)0$.*

The proof of this result is found in the appendix. We will use Diamond and Stiglitz's theorem to prove the following proposition.

Proposition 1. *Let $q^*(r)$ be the optimal level of output that maximizes $U(q)$ in (2). If increases in r represent mean preserving increases in risk, then the level of optimal output q^* increases (decreases) with r if $g(p) = (p - C'(q^*))u'(\pi(p, q^*))$ is strictly convex (concave) function of p. Also, this result is related to a modified version of the relative prudence index.*

Proof. Straightforward calculations show that:

$$g''(p) = (p - C'(q^*))u'''(\pi(p, q^*))q^2 + 2u''(\pi(p, q^*))q \tag{6}$$

Using Diamond and Stiglitz's theorem we can affirm that $dq^*/dr > (<)0$ if and only if $g''(p) > (<)0$. Note that unlike Lippman and McCall (1981), the condition that determines the threshold to have optimal output increases (decreases) in the face of mean-preserving increases in risk is that $g(t) = t'u'(t)$ be convex (concave), where $t \equiv \pi(p, q^*)$ and $t' = p - C'(q^*)$.

Now observe that $g''(p) > (<)0$ is equivalent to

$$H(\pi) \equiv (\pi - b)P(\pi) > (<)2 \tag{7}$$

where $P(\pi) = -u'''(\pi)/u''(\pi)$ is the absolute prudence index, with $b = q(C'(q) - \frac{C(q)}{q})$. The expression $H(\pi)$ is a modified version of the relative prudence index, which is a special case of the index defined by Choi et al. (2001). □

The theoretical foundation of proposition 1 can be derived by comparing the solution of the optimal level of q under certainty to the solution under uncertainty.

When the uncertain price p is replaced by its expectation, the maximization problem turns into

$$\max_{q>0} \hat{U}(q) = u(\mu_\pi(\mu_p, q)) \tag{8}$$

where $\mu_\pi(\mu_p, q) = \mu_p q - C(q)$

The first order condition of this problem is:

$$\hat{U}'(\hat{q}) = (\mu_p - C'(\hat{q}))u'(\mu_\pi(\mu_p, \hat{q})) = 0 \tag{9}$$

Let $\hat{q}$ denote the solution to this maximization problem, i.e. the optimal output under certainty. We want to determine whether the optimal output under uncertainty is greater (less) than the optimal output under certainty. Since $\hat{U}(q)$ is concave in q, we know that $q^* > (<)\hat{q}$ if and only if $\hat{U}'(q^*) < (>)0$. In other words, the optimal output will increase (decrease) under uncertainty whenever:

$$\hat{U}'(q^*) = (\mu_p - C'(q^*))u'(\mu_\pi(\mu_p, q^*)) < (>)0 \tag{10}$$

by replacing (4) in the right hand side of (10) we get:

$$(\mu_p - C'(q^*))u'(\mu_\pi(\mu_p, q^*)) = g(\mu_p) < (>)0 = E(p - C'(q^*))u'(\pi(p, q^*)) = Eg(p) \tag{11}$$

and by Jensen´s inequality we know that equation (11) holds if the function $g(p)$ is convex (concave) in p. This result is precisely the one obtained in Proposition 1 above.

3 The competitive firm under uncertainty in the (μ, σ)-space

Hawawini (1978) was the first to represent the behavior of a firm under uncertainty using the (μ, σ) approach. As shown by Meyer (1987), this framework is equivalent to the expected utility approach if one restricts attainable lotteries to a linear distribution class. In this section of the paper we want to show that even in the case when the competitive firm is represented using the parametric (μ, σ) model, if the firm experiences a mean-preserving increase in risk, the previously determined conditions for an increase or decrease in the optimal output are still valid. In this section we also connect our results with the elasticity of risk aversion developed by Batterman et al. (2002).

Following Meyer (1987), the expected utility from the random variable $Y(\theta, \alpha)$ for an agent with a utility function $u(\cdot)$ can be written in terms of the mean and standard deviation of $Y(\theta, \alpha)$ as:

$$Eu(Y(\theta, \alpha)) = \int_a^b u(\mu_y(\mu_\theta, \alpha) + \sigma_y(\sigma_\theta, \alpha)\epsilon)dF(\epsilon)d\epsilon \equiv V(\mu_y(\mu_\theta, \alpha), \sigma_y(\sigma_\theta, \alpha)) \quad (12)$$

The variable ϵ is a random variable obtained from the normalization of $Y(\theta, \alpha)$, that is $\epsilon = \frac{(Y(\theta,\alpha) - \mu_y(\mu_\theta,\alpha))}{\sigma_y(\sigma_\theta,\alpha)}$, where $\mu_y(\mu_\theta, \alpha)$ and $\sigma_y(\sigma_\theta, \alpha)$ are the mean and standard deviation of $Y(\theta, \alpha)$. Also, α is a control variable and θ is a random variable with mean and standard deviation given by μ_θ and σ_θ, respectively. It is obvious that $\frac{\partial \sigma_y}{\partial \sigma_\theta} > 0$ because σ_θ represents the shift parameter from Diamond and Stiglitz (1974) and therefore, an increase in σ_θ is seen as a mean-preserving spread, which is an increase in σ_y. Finally, we will assume that σ_θ and α are positively related $\frac{\partial \sigma_y}{\partial \alpha} > 0$ and have a complementary effect on risk, that is $\frac{\partial \sigma_y^2}{\partial \sigma_\theta \partial \alpha} > 0$. These three assumptions will become completely clear in section 3.1 in which a we apply these concepts to specific functional forms.

Here we will interpret $Y(\theta, \alpha)$ as the profit of the competitive firm while θ will represent the price uncertainty. In this context, Meyer establishes the following equivalences between $u(\cdot)$ and $V(\cdot,\cdot)$

$$V_\sigma(\mu_y,\sigma_y) = \int_a^b u'(\mu_y+\sigma_y\epsilon)\epsilon dF(\epsilon) < 0 \tag{13a}$$

$$V_\mu(\mu_y,\sigma_y) = \int_a^b u'(\mu_y+\sigma_y\epsilon) dF(\epsilon) > 0 \tag{13b}$$

$$V_{\mu\mu}(\mu_y,\sigma_y) = \int_a^b u''(\mu_y+\sigma_y\epsilon) dF(\epsilon) < 0 \tag{13c}$$

$$V_{\mu\sigma}(\mu_y,\sigma_y) = \int_a^b u''(\mu_y+\sigma_y\epsilon)\epsilon dF(\epsilon) > 0 \tag{13d}$$

$$V_{\sigma\sigma}(\mu_y,\sigma_y) = \int_a^b u''(\mu_y+\sigma_y\epsilon)\epsilon^2 dF(\epsilon) < 0 \tag{13e}$$

where the subscripts denote the partial derivatives, cross derivatives and second derivatives respectively.

Let $R(\mu,\sigma)$ denote the slope of the derived utility function $V(\cdot,\cdot)$, that is $R(\mu,\sigma) = -\frac{V_\sigma}{V_\mu}$. Since $R(\mu,\sigma)$ is the marginal rate of substitution between μ and σ, the literature interprets it as a measure of risk aversion in the (μ,σ) space, which is the analogue to the absolute risk aversion measure in the expected utility framework (see Lajeri, and Nielsen, 2000). We will need the following definition for what follows.

Definition 1 (Elasticity of Risk Aversion, Batterman et al., 2002)**.** *Let $\epsilon_{R,\sigma}$ denote the elasticity of risk aversion with respect to the standard deviation of wealth. Then,*

$$\epsilon_{R,\sigma}(\mu,\sigma) = R_\sigma \frac{\sigma}{R(\mu,\sigma)} \tag{14}$$

where $R_\sigma = \frac{\partial R}{\partial \sigma}$. The elasticity of risk aversion $\epsilon_{R,\sigma}(\mu,\sigma)$ indicates the percentage change in risk aversion over the percentage change in the standard deviation of wealth. We will use this definition in proposition 2.

Now, going back to our optimization problem and assuming that $V(\mu_y(\mu_\theta,\alpha),\sigma_y(\sigma_\theta,\alpha))$ is strictly concave, the first and second order conditions of (12) are:

$$V_\alpha = V_\mu \frac{\partial \mu_y}{\partial \alpha} + V_\sigma \frac{\partial \sigma_y}{\partial \alpha} = 0 \tag{15}$$

$$V_{\alpha\alpha} = V_{\mu\mu}(\frac{\partial \mu_y}{\partial \alpha})^2 + 2V_{\mu\sigma}\frac{\partial \mu_y}{\partial \alpha}\frac{\partial \sigma_y}{\partial \alpha} + V_{\mu}\frac{\partial^2 \mu_y}{\partial \alpha^2} + V_{\sigma\sigma}(\frac{\partial \sigma_y}{\partial \alpha})^2 + V_{\sigma}\frac{\partial^2 \sigma_y}{\partial \alpha^2} < 0 \tag{16}$$

Proposition 2. *Let $\alpha^*(\sigma_\theta)$ be the level of the control variable that maximizes $V(\mu_y(\mu_\theta, \alpha), \sigma_y(\sigma_\theta, \alpha))$ in (12). Then, the following four statements are equivalent:*

(i) α^* increases (decreases) whenever the function $V_\alpha = V_\mu \frac{\partial \mu_y}{\partial \alpha} + V_\sigma \frac{\partial \sigma_y}{\partial \alpha}$ is increasing (decreasing) in σ_θ.

(ii) α^* increases (decreases) if and only if

$P(Y(\theta, \alpha)) > (<)(\frac{\partial \sigma_y}{\partial \alpha}\frac{\partial \sigma_y}{\partial \sigma_\theta} + \sigma_y \frac{\partial^2 \sigma_y}{\partial \sigma_\theta \partial \alpha})/(\sigma_y \frac{\partial Y(\theta,\alpha)}{\partial \alpha}\frac{\partial \sigma_y}{\partial \sigma_\theta})$,

where $P(Y(\theta, \alpha)) = -u'''(Y(\theta, \alpha))/u''(Y(\theta, \alpha))$ is absolute prudence.

(iii) α^* increases (decreases) if and only if

$\epsilon_{R,\sigma_y} < (>) - \sigma_y \frac{\partial^2 \sigma_y}{\partial \alpha \partial \sigma_\theta} / \frac{\partial \sigma_y}{\partial \alpha}\frac{\partial \sigma_y}{\partial \sigma_\theta}$

(iv) α^* increases (decreases) if and only if

$\epsilon_{R,\sigma_\theta} < (>) - \sigma_\theta \frac{\partial^2 \sigma_y}{\partial \alpha \partial \sigma_\theta} / \frac{\partial \sigma_y}{\partial \alpha}$

where ϵ_{R,σ_i} is the elasticity of risk aversion with respect to σ_i.

Proof. Implicitly differentiating the first order condition results in:

$$\frac{d\alpha^*}{d\sigma_\theta} = -\frac{V_{\alpha\sigma_\theta}}{V_{\alpha\alpha}} \tag{17}$$

Since the denominator corresponds to the second order condition in (16) and is therefore negative, $\frac{d\alpha^*}{d\sigma_\theta}$ has the same sign as $V_{\alpha\sigma_\theta}$, which proves part (i) of proposition 2.

Differentiating V_α with respect to σ_θ yields:

$$V_{\alpha\sigma_\theta} = V_{\mu\sigma}\frac{\partial \mu_y}{\partial \alpha}\frac{\partial \sigma_y}{\partial \sigma_\theta} + V_{\sigma\sigma}\frac{\partial \sigma_y}{\partial \alpha}\frac{\partial \sigma_y}{\partial \sigma_\theta} + V_\sigma \frac{\partial^2 \sigma_y}{\partial \sigma_\theta \partial \alpha} \tag{18}$$

And if we replace (13a), (13d) and (13e) in (18) we get:

$$\begin{gathered}\int_a^b u''(Y(\theta,\alpha))\epsilon dF(\epsilon)[\tfrac{\partial\mu_y}{\partial\alpha}\tfrac{\partial\sigma_y}{\partial\sigma_\theta}] + \int_a^b u''(Y(\theta,\alpha))\epsilon^2 dF(\epsilon)[\tfrac{\partial\sigma_y}{\partial\alpha}\tfrac{\partial\sigma_y}{\partial\sigma_\theta}] + \\ \int_a^b u'(Y(\theta,\alpha))\epsilon dF(\epsilon)[\tfrac{\partial\sigma_y^2}{\partial\alpha\partial\sigma_\theta}]\end{gathered} \tag{19}$$

which in turn becomes

$$\int_a^b u''(Y(\theta,\alpha)[\frac{\partial Y(\theta,\alpha)}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}] + u'(Y(\theta,\alpha)[\frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha}]\epsilon dF(\epsilon) \tag{20}$$

Now if we apply integration by parts we get:

$$\begin{gathered}[u''(Y(\theta,\alpha))[\tfrac{\partial Y(\theta,\alpha)}{\partial\alpha}\tfrac{\partial\sigma_y}{\partial\sigma_\theta}] + u'(Y(\theta,\alpha))[\tfrac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha}]]\int_a^b \epsilon dF(\epsilon) \\ -\int_a^b(\int_a^\epsilon zdF(z)(u'''(Y(\theta,\alpha))\tfrac{\partial Y(\theta,\alpha)}{\partial\alpha}\tfrac{\partial\sigma_y}{\partial\sigma_\theta}\sigma_y + u''(Y(\theta,\alpha))(\tfrac{\partial\sigma_y}{\partial\alpha}\tfrac{\partial\sigma_y}{\partial\sigma_\theta} + \tfrac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha}\sigma_y)))d\epsilon\end{gathered} \tag{21}$$

and since $E(\epsilon) = 0$, condition (21) simplifies to

$$-\int_a^b(\int_a^\epsilon zdF(z)(u'''(Y(\theta,\alpha))[\frac{\partial Y(\theta,\alpha)}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}\sigma_y] + u''(Y(\theta,\alpha))[\frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta} + \frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha}\sigma_y])d\epsilon \tag{22}$$

and given that $\int_a^\epsilon zdF(z) < 0$ for $z < b$, then condition (22) is positive (negative) whenever

$$u'''(Y(\theta,\alpha)[\frac{\partial Y(\theta,\alpha)}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}\sigma_y] + u''(Y(\theta,\alpha))[\frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta} + \frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha}\sigma_y] > (<)0 \tag{23}$$

by rearranging the above expression we obtain

$$\frac{-u'''(Y(\theta,\alpha))}{u''(Y(\theta,\alpha))} = P(Y(\theta,\alpha)) > (<)(\frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta} + \sigma_y\frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha})/(\sigma_y\frac{\partial Y(\theta,\alpha)}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}) \tag{24}$$

which prove part (ii) of proposition 2.

To prove the part (iii) and (iv) of proposition 2 we replace (15) in (18), and rearranging the expression, we get:

$$V_{\alpha\sigma_\theta} = V_\sigma\frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha} + [V_{\mu\sigma}R(\mu,\sigma) + V_{\sigma\sigma}]\frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta} \tag{25}$$

We are interested in the response of α^* to changes in σ_θ. We already know that $\frac{\partial^2\sigma_y}{\partial\sigma_\theta\partial\alpha} \geq 0$ and $\frac{\partial\sigma_y}{\partial\alpha}, \frac{\partial\sigma_y}{\partial\sigma_\theta} > 0$ as they were defined at the beginning of this section. In the context, the reaction of α^*, depends on a direct effect and an indirect effect. The first term in equation (25) is the direct effect, which is negative, while the second is an indirect effect that has an ambiguous sign. To connect our results with the elasticity of risk aversion, let us differentiate $R(\mu_y, \sigma_y)$ with respect to σ_y. At the optimum, the elasticity of risk aversion is given by:

$$\epsilon_{R,\sigma} \equiv R_\sigma \frac{\sigma_y}{R(\mu_y, \sigma_y)} = -\frac{-[V_{\sigma\sigma}V_\mu - V_\sigma V_{\mu\sigma}]}{V_\mu^2} \frac{\sigma_y}{R(\mu_y, \sigma_y)} = \frac{\sigma_y}{V_\sigma}[V_{\sigma\sigma} + R(\mu_y, \sigma_y)V_{\mu\sigma}] \quad (26)$$

if we replace (26) in (25) and rearrange the expression we get

$$V_{\alpha\sigma_\theta} = V_\sigma[\frac{\partial^2 \sigma_y}{\partial\sigma_\theta \partial\alpha} + \frac{\epsilon_{R,\sigma}}{\sigma_y}\frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}] \quad (27)$$

given that $V_\sigma < 0$, $V_{\alpha\sigma_\theta} > (<)0$ if the square bracket is negative (positive). Therefore $\frac{d\alpha^*}{d\sigma_\theta} > (<)0$ if and only if $\epsilon_{R,\sigma} < (>) - \sigma_y \frac{\partial^2\sigma_y}{\partial\alpha\partial\sigma_\theta} / \frac{\partial\sigma_y}{\partial\alpha}\frac{\partial\sigma_y}{\partial\sigma_\theta}$, which proves part (iii) of proposition 2.

Note that the elasticity of risk aversion with respect to σ_θ is given by:

$$\epsilon_{R,\sigma_\theta} = \frac{dR}{d\sigma_\theta}\frac{\sigma_\theta}{R} = \frac{\partial R}{\partial\sigma_y}\frac{\sigma_y}{R}\frac{\sigma_\theta}{\sigma_y}\frac{\partial\sigma_y}{\partial\sigma_\theta} = \epsilon_{R,\sigma_y}\frac{\sigma_\theta}{\sigma_y}\frac{\partial\sigma_y}{\partial\sigma_\theta} \quad (28)$$

to obtain part (iv) of proposition 2 by replacing (28) in (27). □

3.1 An application with a specific functional form

Now, we will go back to our firm under price uncertainty application, but this time in the mean-variance space.

Let $\mu_\pi(\mu_p, q) = \mu_p q - C(q)$ and $\sigma_\pi(\sigma_p, q) = \sigma_p q$, then

$$\max_{q>0} V(\mu_\pi(\mu_p, q), \sigma_\pi(\sigma_p, q)) \quad (29)$$

then the first and second order conditions are:

$$V_q = V_\mu(\mu_p - C'(q^*)) + V_\sigma \sigma_p \tag{30}$$

$$V_{qq} = V_{\mu\mu}(\mu_p - C'(q^*))^2 + 2V_{\mu\sigma}(\mu_p - C'(q^*))\sigma_p + V_\mu(-C''(q^*)) + V_{\sigma\sigma}(\sigma_p)^2 < 0 \tag{31}$$

Proposition 3. *Let $q^*(\sigma_p)$ be the level of output that maximizes $V(\mu_\pi(\mu_p, q), \sigma_\pi(\sigma_p, q))$ in (28). Then, the following two statements are equivalent:*

(i) q^* increases (decreases) if and only if $(\pi - b)P(\pi(p,q)) > (<)2$, where $b = q(C'(q) - \frac{C(q)}{q})$.

(ii) q^* increases (decreases) if and only if $\epsilon_{R,\sigma_i} < (>) - 1$.

Proof. We note that:

$$\frac{\partial \sigma_\pi}{\partial q} = \sigma_p; \frac{\partial \sigma_\pi}{\partial \sigma_p} = q; \frac{\partial^2 \sigma_\pi}{\partial \sigma_p \partial q} = 1. \tag{32}$$

If we replace (32) in part (ii) of the proposition 2 and rearranging the expression we get

$$q(\mu_p - C'(q))P(\pi(p,q)) > (<)2 \tag{33}$$

which is exactly the result obtained in equation (7) of proposition 1. On the other hand, if we replace (32) in parts (iii) and (iv) of proposition 2 we show that the optimal output increases (decreases) if and only if the elasticity of risk aversion is less (bigger) than -1. □

4 Utility Premium and the competitive firm under price uncertainty

The utility premium simply measures the loss of utility from consuming the random quantity $\pi(p, q)$ instead of the certain quantity $\mu_\pi(\mu_p, q)$. Eeckhoudt and Schlesinger (2009) refer to this concept as an intra-personal measure of pain, where pain is measured via a decrease in the expected utility.

In order to show the relationship between the output of the competitive firm and the utility premium of Friedman and Savage (1948), we need to rewrite the consumer maximization problem in the following way:

$$\max_{q>0} U(q) = u(\mu_\pi(\mu_p, q)) - \phi(\pi(p, q)) \tag{34}$$

where $\phi(\pi(p, q)) = u(\mu_\pi(\mu_p, q)) - EU(\pi(p, q))$ is the utility premium.

Proposition 4. *The optimal output under uncertainty $q^*(\sigma_p)$ is greater (lesser) than the optimal output under certainty $\hat{q}$, if and only if $\phi'(\pi(p, q))$ is negative (positive).*

Proof. The first-order condition of (34) is:

$$U'(q^*) = (\mu_p - C'(q^*))u'(\mu_\pi(\mu_p, q^*)) - \phi'(\pi(p, q^*)) = 0 \tag{35}$$

where

$$\phi'(\pi(p, q^*)) = (\mu_p - C'(q^*))u'(\mu_\pi(\mu_p, q^*)) - E(p - C'(q^*))u'(\pi(p, q^*))$$ □

Note that equation (35) is equivalent to (4). From (35) we have that $q^* > (<)\hat{q}$ whenever $\phi'(\pi(p, q^*) < (>)0$. This means that an entrepreneur can reduce the pain of the risky price by increasing the output a bit if the utility premium is decreasing in the output. The precautionary output is this additional output that reduces the pain from a risky output. Note that from the definition of utility premium we have $\phi'(\pi(p, q^*)) < (>)0$ whenever $E(p - C'(q^*))u'(\pi(p, q^*)) > (<)(\mu_p - C'(q^*)u'(\mu_\pi(\mu_p, q^*)))$. And this result is precisely the one obtained in equation (11) above.

5 Conclusions

We have provided a new look into the theory of the competitive firm under price uncertainty. We study the conditions under which a mean-preserving increase in risk translates into changes in the level of optimal output (increases or decreases). We have connected these results with the classic mean-variance model of the competitive firm under uncertainty. In the process of doing so, we have shown how our results can be linked with recent advances in the economic theory of risk, like a modified version of the relative prudence index of Choi et al. (2001) and the elasticity of risk aversion of Batterman et al. (2002). Finally, we have connected our results with the classic idea of the utility premium of Friedman and Savage (1948) and we have provided the conditions under which a mean preserving increase in risk in this context translates into changes in optimal output, providing a fresh economic intuition of this classic result. Finally, we have shown that the conditions to sign the effect of price uncertainty on firm´s output are the same, whenever we work in the expected utility model or in the (μ, σ) framework.

A Appendix

Proof of Theorem 1 Diamond and Stiglitz (1974)

Theorem 1: Let $\alpha^*(r)$ be the level of the control variable that maximizes $\int_a^b u(\theta, \alpha) dF(\theta, r)$. If increases in r represent mean preserving increases in risk, then α^* increases (decreases) with r if u_α is strictly convex (concave) function of θ, i.e., if $u_{\alpha\theta\theta} > (<) 0$.

Proof

First, instead of using the classic notation of $dF(\theta, r)$ to denote the integrand, we will be using $F_\theta d\theta$.

Definition: An increase in r represents a mean preserving increase in risk if $\int_0^1 F_r(\theta, r) = 0$, and also $T(y, r) = \int_0^y F_r(\theta, r) d\theta \geq 0$ for $0 \leq y \leq 1$.

The decision maker has to choose α in order to maximize

$$\int u(\theta, \alpha) F_\theta d\theta$$

The first order condition of the above problem is

$$\int u_\alpha(\theta, \alpha) F_\theta d\theta = 0 = H(\alpha^*, r, \theta)$$

and by applying the implicit function theorem we get

$$\frac{d\alpha^*}{dr} = \frac{-H_r}{H_\alpha} = \frac{-\int u_\alpha(\theta, \alpha) F_{\theta r} d\theta}{\int u_{\alpha\alpha}(\theta, \alpha) F_\theta d\theta}$$

We know that $\int u_{\alpha\alpha}(\theta, \alpha) F_\theta d\theta$ corresponds to the second order conditions of the original problem, and is therefore negative. Consequently, the sign of $\frac{d\alpha^*}{dr}$ depends only on the sign of

$$\int u_\alpha(\theta, \alpha) F_{\theta r} d\theta$$

if we integrate by parts to the previous equation we obtain

$$-\int u_{\alpha\theta}(\theta, \alpha) F_r d\theta$$

if we integrate by parts a second time to the above expression we get

$$\int u_{\alpha\theta\theta}(\theta, \alpha)T(\theta, r)d\theta$$

Finally, since we already know that $T(\theta, r) \geq 0$, then the sign of $\frac{d\alpha^*}{dr}$ depends only on the sign of $u_{\alpha\theta\theta}(\theta, \alpha)$. We can thus conclude that if $u_{\alpha\theta\theta}(\theta, \alpha)T(\theta, r) > (<)0$, then $\frac{d\alpha^*}{dr} > (<)0$.

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